Dynamic mortality tables in the UK

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CNSF Insurance Seminar, Mexico City
Friday 25 November 2005
Mortality in England & Wales has generally been improving over last 150 years.
England & Wales, males, 1838-2002, mortality rates

Rate per 1,000

Year


85+
75-84
65-74
55-64
45-54
35-44
25-34
20-24
15-19
10-14
Assurances:

Using older tables is on the safe side for life offices

Annuities and pensions:

Using older tables is unsafe for life offices
Use projected (forecast) mortality
Projected tables give rates:
for each future calendar year
or (diagonally) for each year of birth

Life assurance companies usually use
year of birth tables for premium
calculation

Pension funds usually use future calendar
year tables for valuation
Mortality tables in UK produced by:
Continuous Mortality Investigation Bureau (CMI)
Started by actuarial bodies in 1924
still going strong
Mortality data analysed in 4-year periods

New tables produced every 12 years or so:

1967-70 (1968 base)
1979-82 (1980 base)
1999-2002 (2000 base) under construction
Pensioners (those drawing pension annuities from pension schemes insured with life offices)
Males and females separately
“Lives” and “Amounts” analysed

PMA Tables:
Pensioners Males Amounts
Also PML, PFA, PFL
1967-70 base called PMA68 Base

1979-82 base called PMA80 Base

1991-94 base called PMA92 Base

PMA68Base
PMA80Base
PMA92Base
Bigger improvements at younger pensioner ages, below 70

Apparent improvements above 100 just a feature of curve fitting method
1968 base
Projected as 1 year reduction of age every 20 calendar years
Table projected to calendar year 1990, 22 years ahead
Called PA(90) or (by me) PMA68C1990
Also year of birth tables:
e.g. PMA68B1930

- PMA68Base
- C1990
- C2010
- C2030
- C2050

PMA68Base
B1930
B1950
B1970
1980 base

Projection by formula:

$q(x,t)$ for age $x$, calendar year $t$

$q(x,t) = q(x,0) \times RF(x,t)$

$RF(x,t) = \alpha(x) + [1 - \alpha(x)] \times [1 - f_n]^{t/n}$

Choose:

$n = 20$

$f_{20} = 0.6$
\[ \alpha(x) = \begin{cases} 0.5 & x < 60 \\ \frac{(x - 10)}{100} & 60 \leq x \leq 110 \\ 1 & x > 110 \end{cases} \]

\[ RF(x,20) = 0.4 + 0.6 \alpha(x) \]

\[ RF(x,20) = \begin{cases} 0.7 & x < 60 \\ \frac{(0.6x + 34)}{100} & 60 \leq x \leq 110 \\ 1 & x > 110 \end{cases} \]

- PMA80Base
- C1990
- C2010
- C2030
- C2050

Graph showing the relationship between age and the probability density function $q(x)$. The graph compares the following lines:

- **PMA80Base**
- **B1930**
- **B1950**
- **B1970**

The graph plots age on the horizontal axis and the probability density function on the vertical axis.
Bigger reductions than 1968 projection
(1 year in 20)
Now about
1 year in 6 at age 65
1 year in 9 at age 90
1992 base

Again by formula:

\[ RF(x,t) = \alpha(x) + [1 - \alpha(x)] \times [1 - f(x)]^{t/20} \]

\[ f(x) = h = 0.55 \quad x < 60 \]

\[ = [ (110 - x) \times h + (x - 60) \times k ] / 50 \quad 60 \leq x \leq 110 \]

\[ = k = 0.29 \quad x > 110 \]
\[ \alpha(x) = c = 0.13 \quad x < 60 \]
\[ = 1 + (1 - c) \times (x - 110) / 50 \quad 60 \leq x \leq 110 \]
\[ = 1 \quad x > 110 \]

More complicated than 1980 method

\( q(x) \)

- **PMA92Base**
- **B1930**
- **B1950**
- **B1970**
1992 base: calendar years 2010, 2030, 2050

- **PMA92Base**
- **C2010**
- **C2030**
- **C2050**
Similar reductions to 1980 projection

1 year in 6 at age 65
1 year in 9 at age 90

Now about

1 year in 5 at age 65
1 year in 9 at age 90

- PMA68B1950
- PMA80B1950
- PMA92B1950
Age 65: 1968 base, 1980 base, 1992 base
Age 90: 1968 base, 1980 base, 1992 base

Graph showing the decline in mortality rates (q(x)) for individuals aged 90 from 1960 to 2050. The graph compares three scenarios based on different base years: 1968, 1980, and 1992. The data shows a consistent decrease in mortality rates over time for all scenarios, with the 1968 and 1980 bases starting higher and the 1992 base starting lower.

Legend:
- PMA68
- PMA80
- PMA92
Annuity values $a(x)$ at 4%

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>PMA68Base</td>
<td>9.51</td>
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<tr>
<td>PMA80Base</td>
<td>10.03</td>
</tr>
<tr>
<td>PMA92Base</td>
<td>11.22</td>
</tr>
</tbody>
</table>
Age 65 in base year:

PMA68B1903    9.66    (cf 9.51)
PMA80B1915    10.47    (cf 10.03)
PMA92B1927    11.79    (cf 11.79)
<table>
<thead>
<tr>
<th>Born:</th>
<th>1930</th>
<th>1950</th>
<th>1970</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMA68</td>
<td>10.13</td>
<td>10.48</td>
<td>10.82</td>
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<td>PMA80</td>
<td>11.02</td>
<td>11.39</td>
<td>11.54</td>
</tr>
<tr>
<td>PMA92</td>
<td>11.96</td>
<td>12.83</td>
<td>13.35</td>
</tr>
</tbody>
</table>
Projecting
e.g. using 1980Base and 1992Base

Two ideas:
improvements depend only on age
improvements depend on year of birth
1. Take ratio \( r(x) = \frac{q(x,92)}{q(x,80)} \)
   Calculate using ratio for same age:
   \[ q(x,2004) = q(x,92) \times r(x) \]
   \[ q(x,2016) = q(x,2004) \times r(x) \]

2. Calculate using ratio for year of birth:
   \[ q(x,2004) = q(x,92) \times r(x - 12) \]
   \[ q(x,2016) = q(x,2004) \times r(x - 24) \]
Projections based on 1992 and using 1980 in two ways
More modern methods under consideration
e.g. penalised splines (P-splines)

See http://www.actuaries.org.uk
Recent paper in

Also search under “CMI”
Now an emphasis on uncertainty of forecasts
Need “confidence intervals”

Many methods suggested
Lee-Carter
Lee and Yang (different Lee)
P-splines
Andrew Smith
Time series
Example simple model, same for all ages:

\[ q(x,t) = qF(x,t) \times \exp(X(t) - \text{var}[X(t)]/2) \]

\(qF(x,t)\) is central (mean) projection

\(\text{Var}[X(t)]/2\) needed to keep mean unchanged

\(X(t)\) defined in three ways
1/ $X(t)$ is random walk:

$$X(t) = X(t-1) + Z(t)$$

$$Z(t) \sim N(0, \sigma^2)$$

2/ $X(t)$ is sum of autoregressive changes:

$$X(t) = X(t-1) + Y(t)$$

$$Y(t) = \alpha Y(t-1) + Z(t)$$

$$Z(t) \sim N(0, \sigma^2)$$

If $\alpha = 0$ then $Y(t) = Z(t)$ and same as 1
3/ \( X(t) \) is autoregressive:
\[
X(t) = a.X(t-1) + Z(t)
\]
\[
Z(t) \sim N(0,\sigma^2)
\]

If \( a = 0 \) then again same as 1
Projections three ways with 95% confidence intervals

Year ahead

q(x)

AR3 High
AR2 High
RW High
Mean
RW Low
AR2 Low
AR3 Low
Much greater uncertainty with some models than with others

In fact much easier for mortality rates to go up than down:
  wars, infections, natural disasters
So perhaps keeping the mean the same as the projection is not so good
Perhaps keep the median the same
Projections three ways with 95% confidence intervals; retaining median

Year ahead

q(x)

- $AR/3$ High
- $AR/2$ High
- RW High
- Mean
- RW Low
- $AR/2$ Low
- $AR/3$ Low
Still much to be done about stochastic mortality
End