Dynamic mortality tables in the UK

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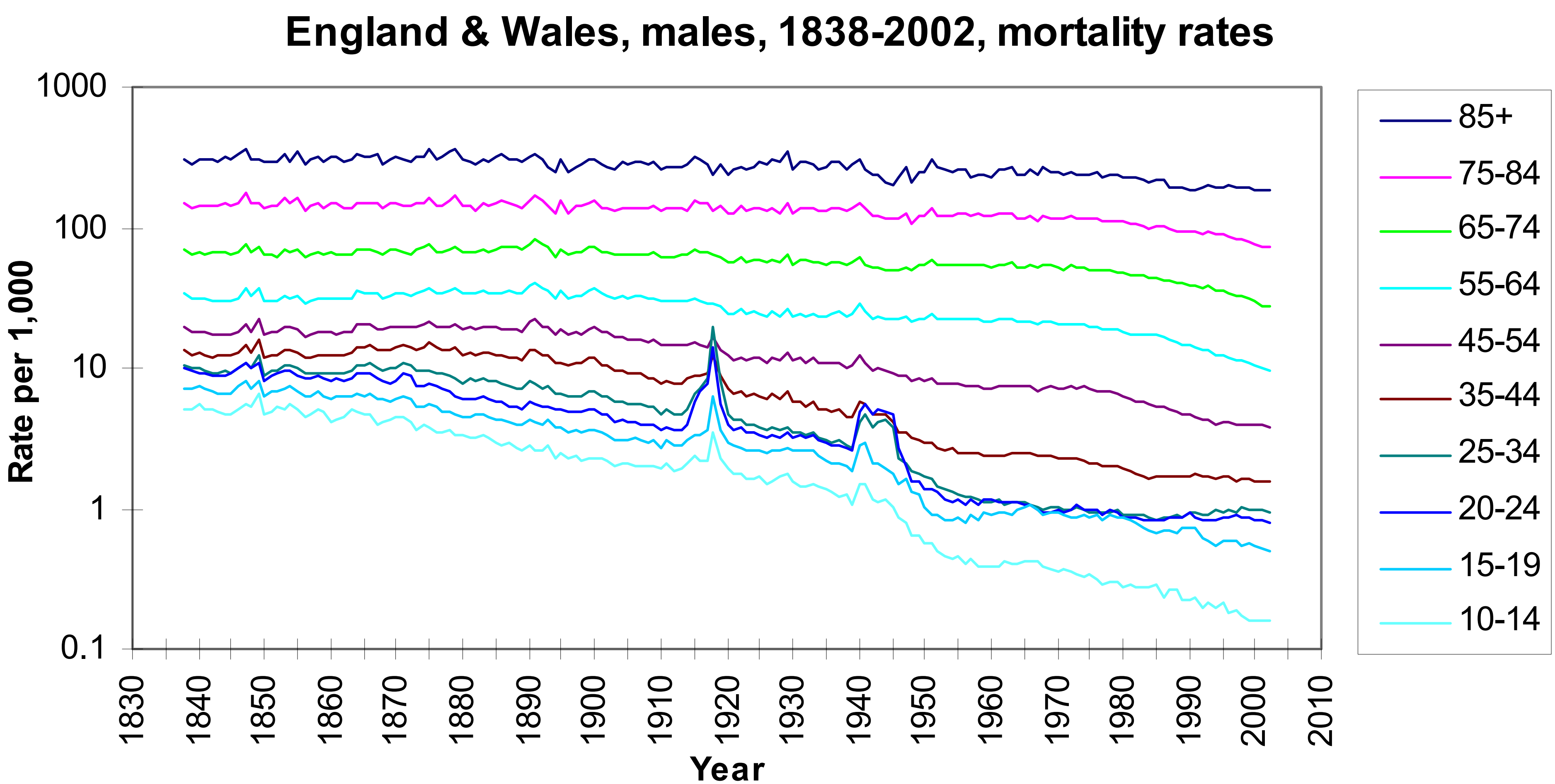
Mortality in England &Wales has generally been improving over last 150 years


England & Wales, males, 1838-2002, mortality rates
Rate per 1,000
1000
100
10
1
0.1
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Year
85+
75-84
65-74
55-64
45-54
35-44
25-34
20-24
15-19
10-14

Assurances:

Using older tables is on the safe side for life offices

Annuities and pensions:

Using older tables is unsafe for life offices

Use projected (forecast) mortality

Projected tables give rates:

for each future calendar year

or (diagonally) for each year of birth

Life assurance companies usually use year of birth tables for premium calculation

Pension funds usually use future calendar year tables for valuation

Mortality tables in UK produced by:

- **Continuous Mortality Investigation Bureau (CMI)**

Started by actuarial bodies in 1924

- **still going strong**

Mortality data analysed in 4-year periods

New tables produced every 12 years or so:

- **1967-70 (1968 base)**
- **1979-82 (1980 base)**
- **1991-94 (1992 base)**
- **1999-2002 (2000 base) under construction**

Pensioners (those drawing pension annuities from pension schemes insured with life offices)

Males and females separately

"Lives" and "Amounts" analysed

PMA Tables:

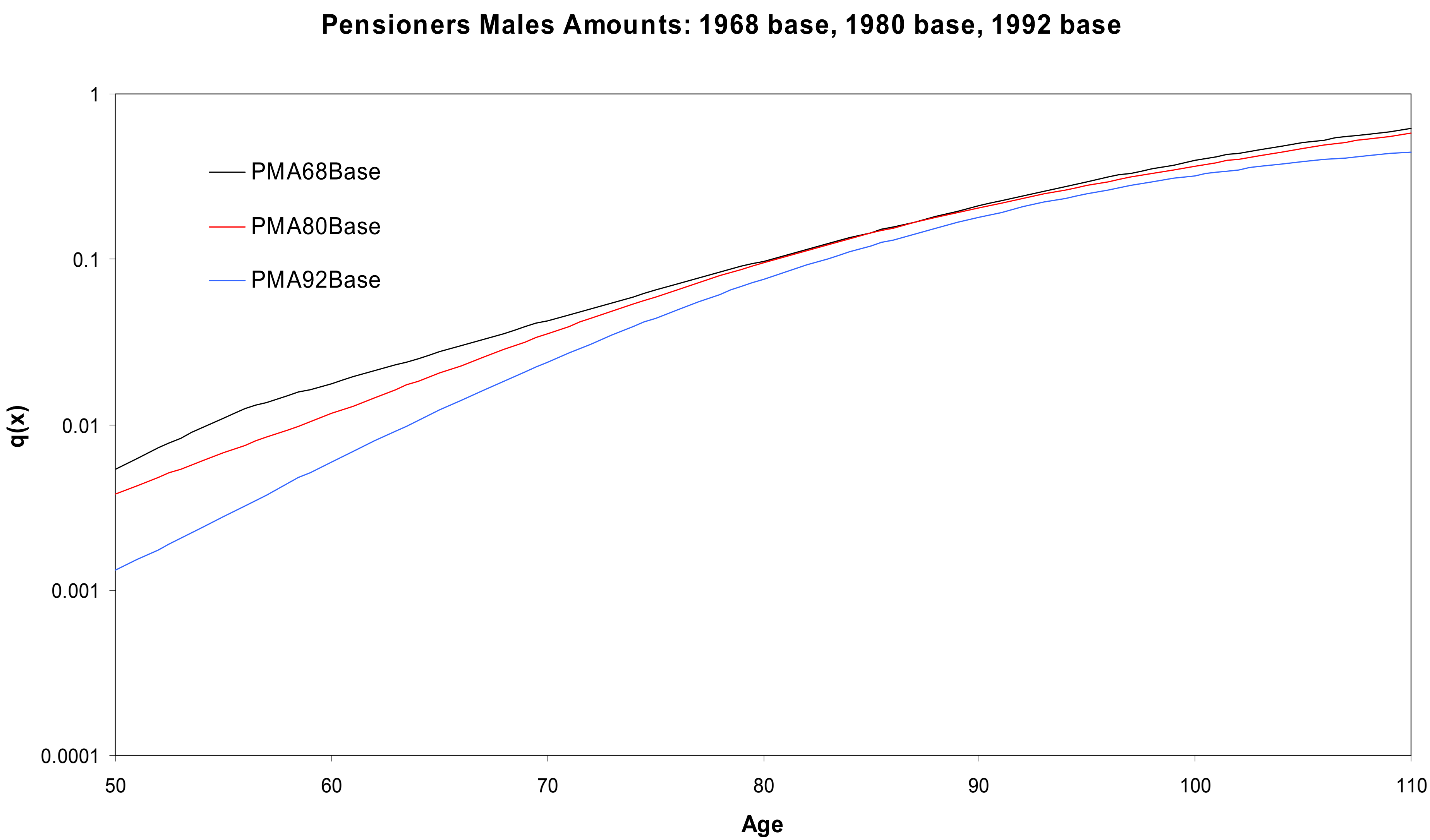
Pensioners Males Amounts

Also PML, PFA, PFL

1967-70 base called PMA68 Base

1979-82 base called PMA80 Base

1991-94 base called PMA92 Base


Pensioners Males Amounts: 1968 base, 1980 base, 1992 base
PMA68Base
PMA80Base
PMA92Base
q(x)
1
0.1
0.01
0.001
0.0001
50
60
70
80
90
100
110
Age

Bigger improvements at younger pensioner ages, below 70

Apparent improvements above 100 just a feature of curve fitting method

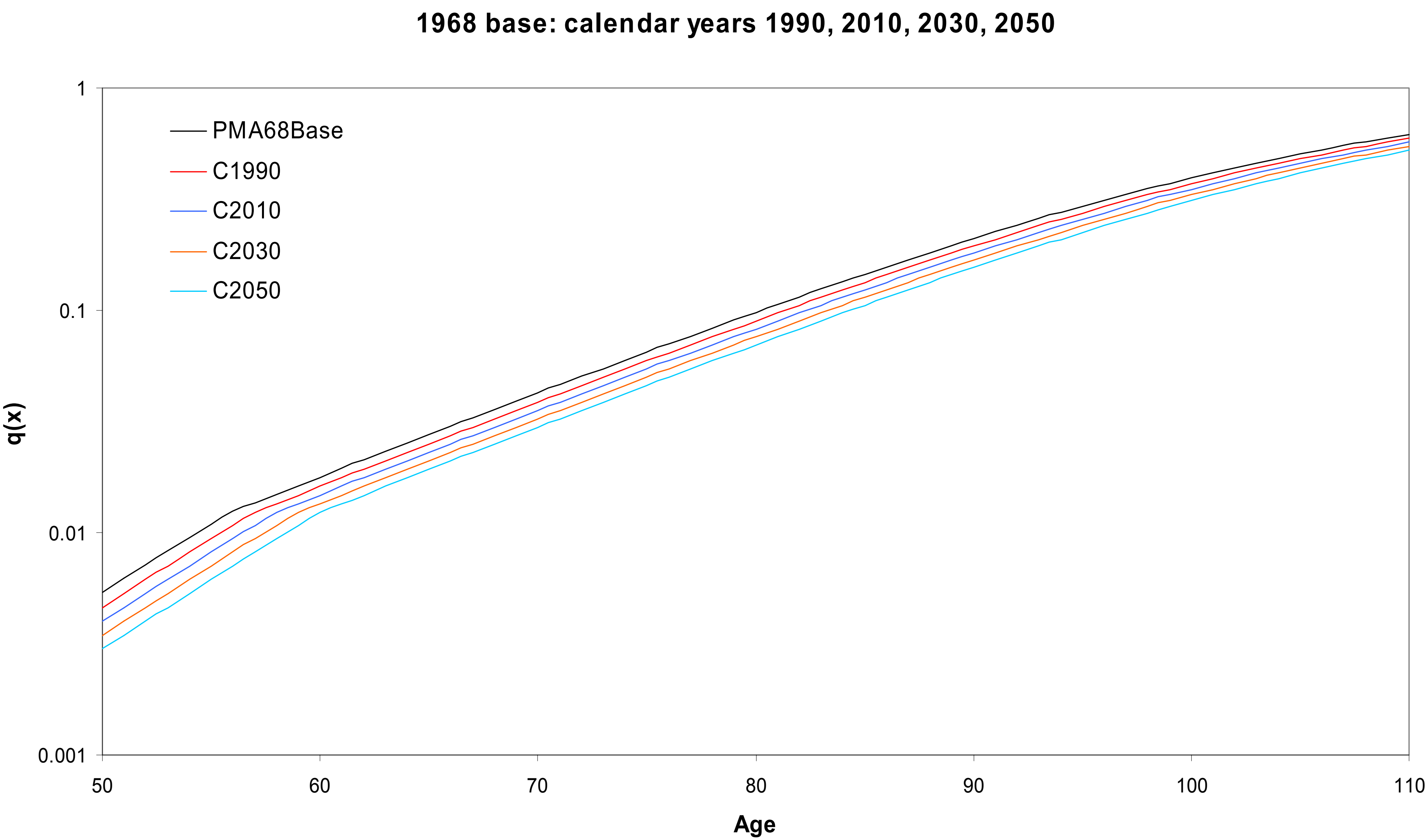
1968 base

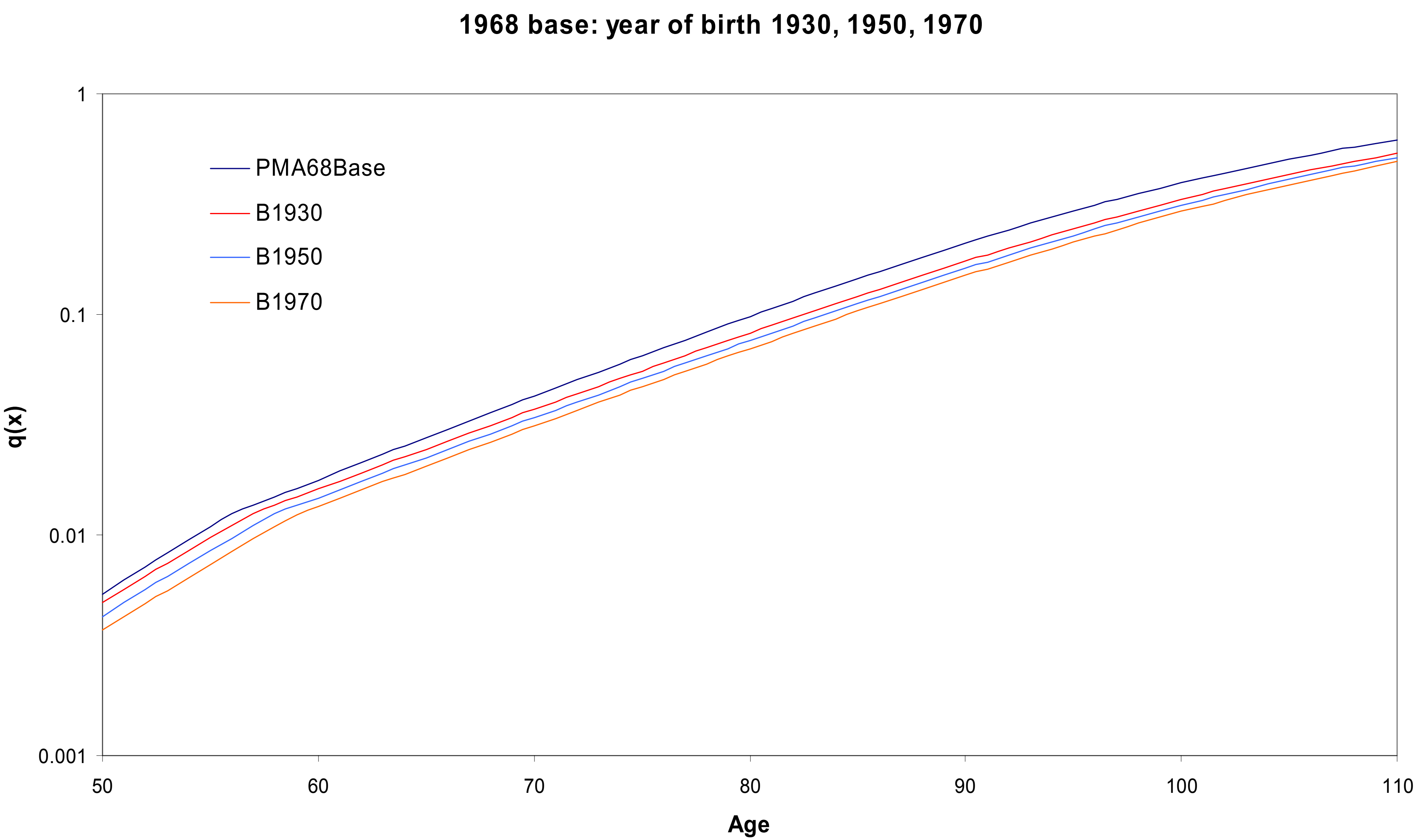
Projected as 1 year reduction of age every 20 calendar years

Table projected to calendar year 1990, 22 years ahead

Called PA(90) or (by me) PMA68C1990

Also year of birth tables: e.g. PMA68B1930


1968 base: calendar years 1990, 2010, 2030, 2050
PMA68Base
C1990
C2010
C2030
C2050
1
0.1
0.01
0.001
q(x)
50
60
70
80
90
100
110
Age


1968 base: year of birth 1930, 1950, 1970
q(x)
Age
1
0.1
0.01
0.001
50
60
70
80
90
100
110
PMA68Base
B1930
B1950
B1970

1980 base

Projection by formula:

q(x,t) for age x, calendar year t

$$q(x,t) = q(x,0) \times RF(x,t)$$

$$RF(x,t) = \alpha(x) + [1 - \alpha(x)] \times [1 - f_n]^{t/n}$$

Choose:

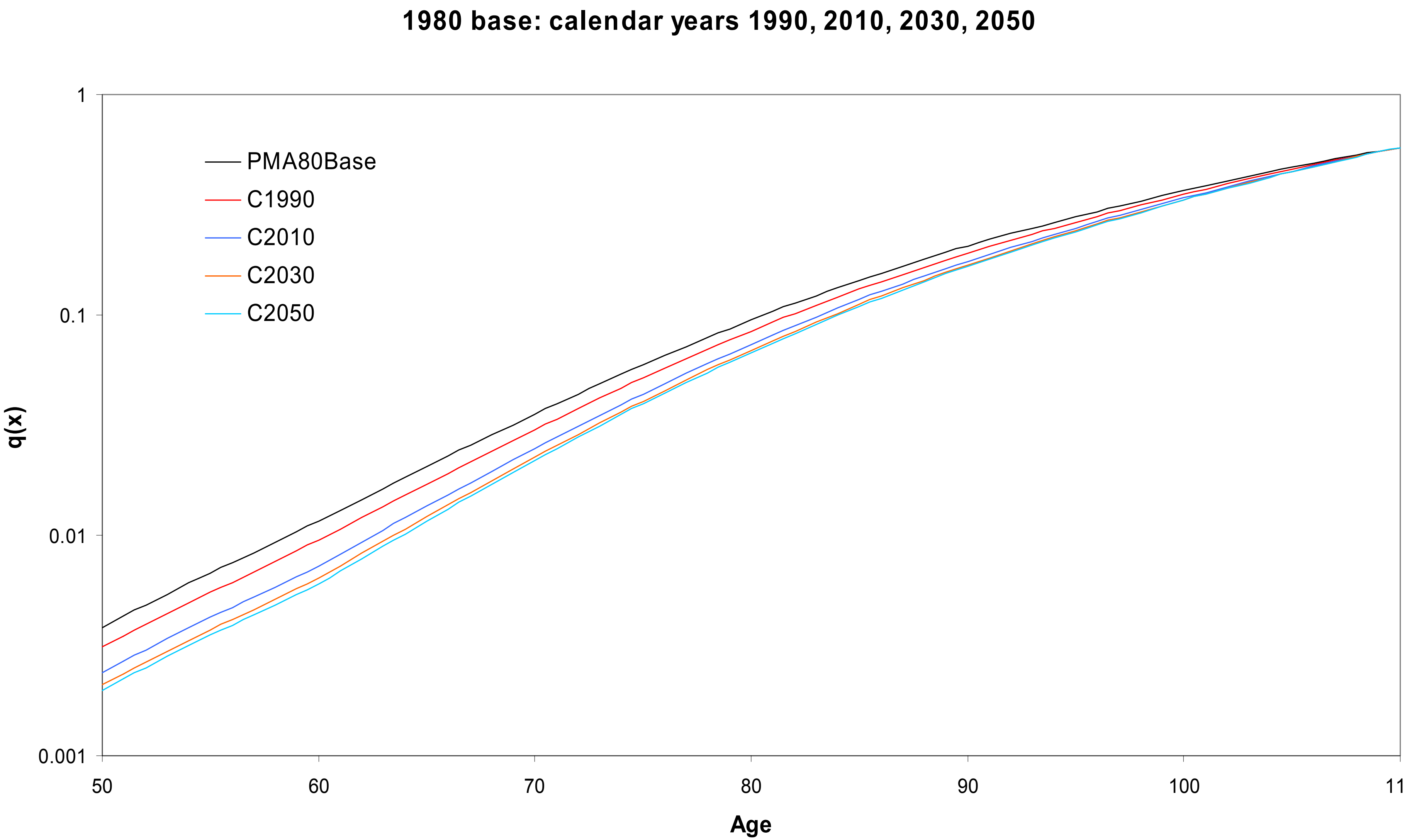
$$n = 20$$

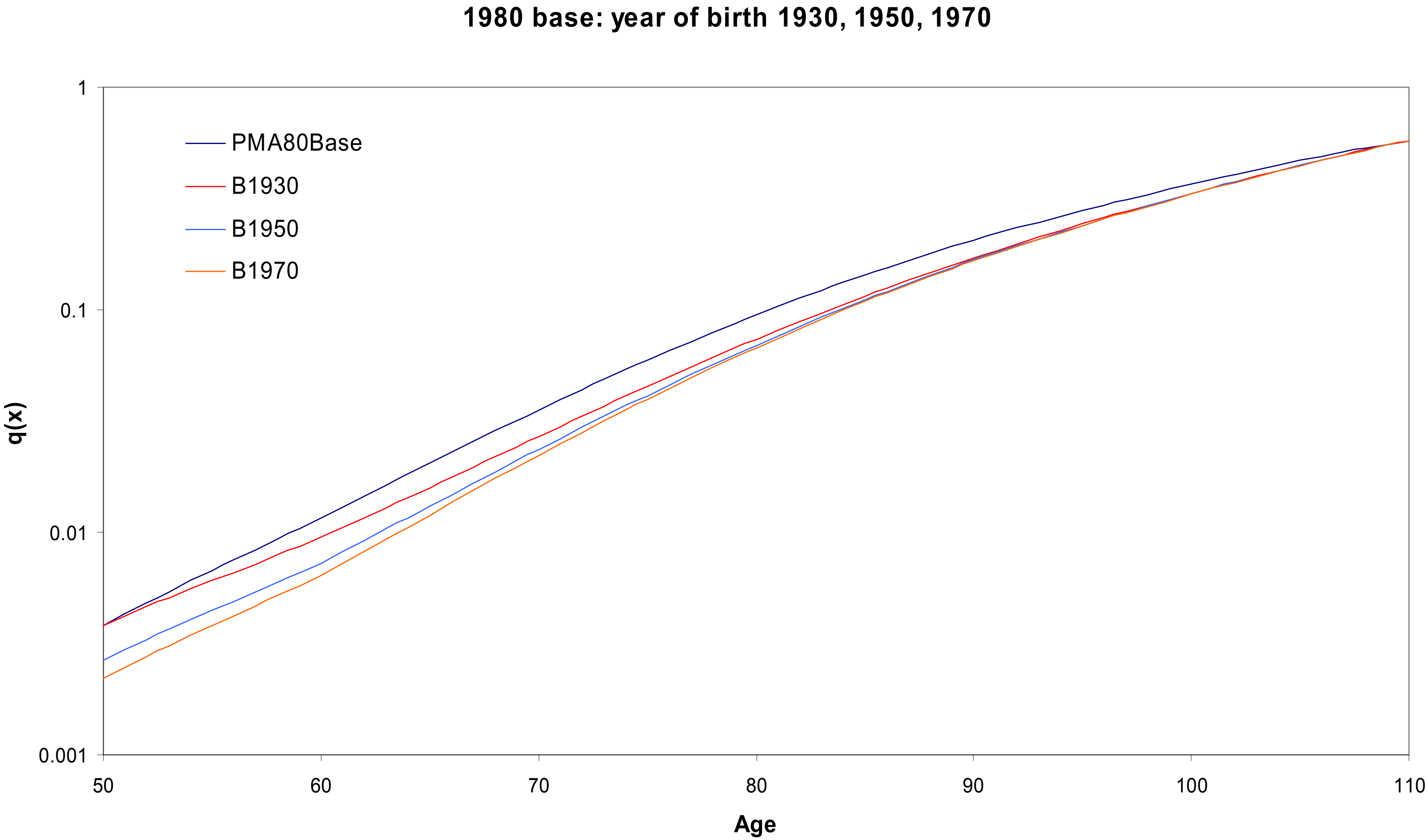
$$f_{20} = 0.6$$

$$\alpha(x) = \begin{cases} 0.5 & x < 60 \\ (x - 10)/100 & 60 \le x \le 110 \\ 1 & x > 110 \end{cases}$$

$$RF(x,20) = 0.4 + 0.6\,\alpha(x)$$

$$RF(x,20) = \begin{cases} 0.7 & x < 60 \\ (0.6x + 34)/100 & 60 \le x \le 110 \\ 1 & x > 110 \end{cases}$$


1980 base: calendar years 1990, 2010, 2030, 2050
PMA80Base
C1990
C2010
C2030
C2050
q(x)
1
0.1
0.01
0.001
50
60
70
80
90
100
110
Age


1980 base: year of birth 1930, 1950, 1970
PMA80Base
B1930
B1950
B1970
1
0.1
0.01
0.001
q(x)
50
60
70
80
90
100
110
Age

Bigger reductions than 1968 projection

(1 year in 20)

Now about

1 year in 6 at age 65

1 year in 9 at age 90

1992 base

Again by formula:

$$RF(x,t) = \alpha(x) + [1 - \alpha(x)] \times [1 - f(x)]^{t/20}$$

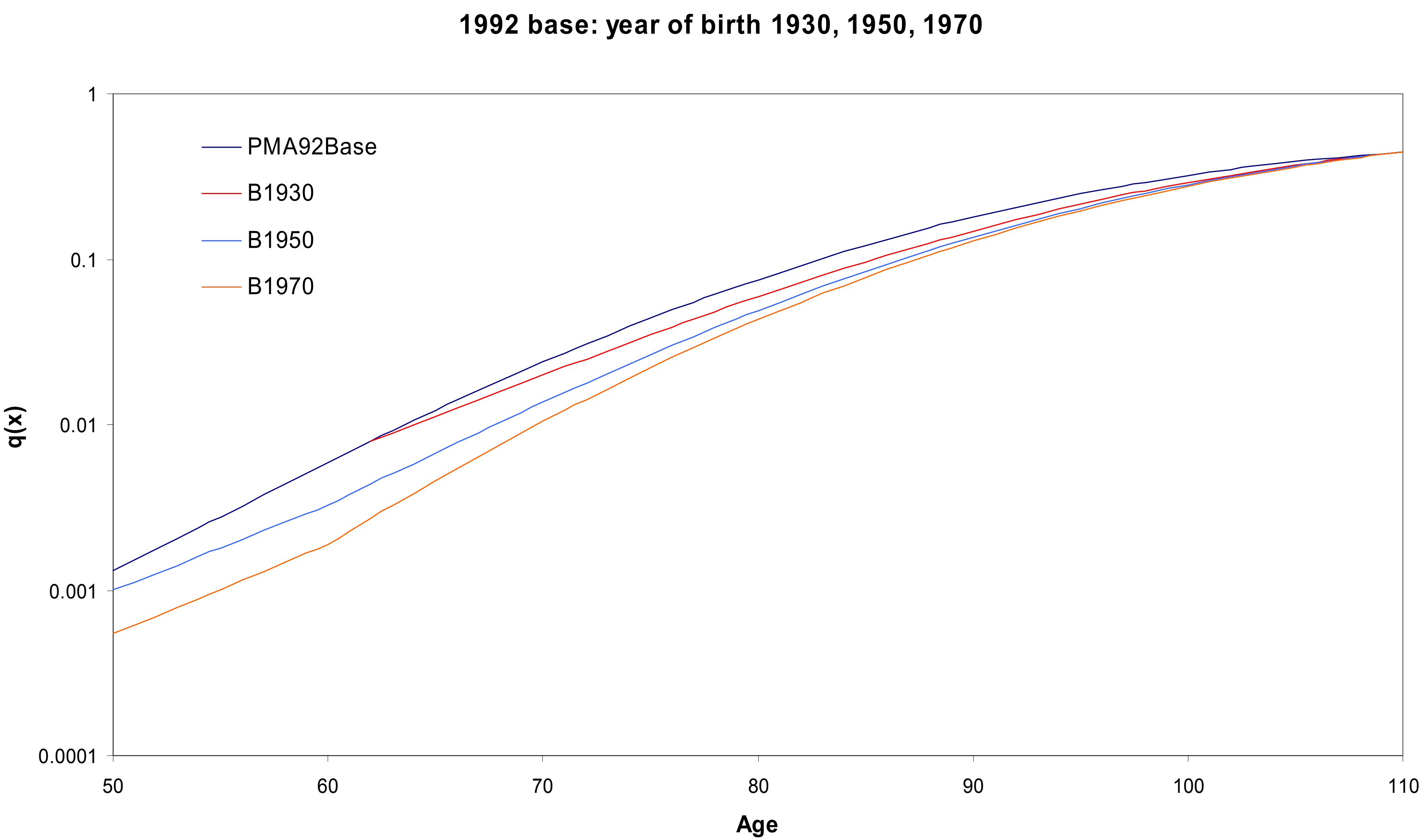
$$f(x) = h = 0.55 \qquad x < 60$$

$$= [(110 - x) \times h + (x - 60) \times k] / 50 \qquad 60 \le x \le 110$$

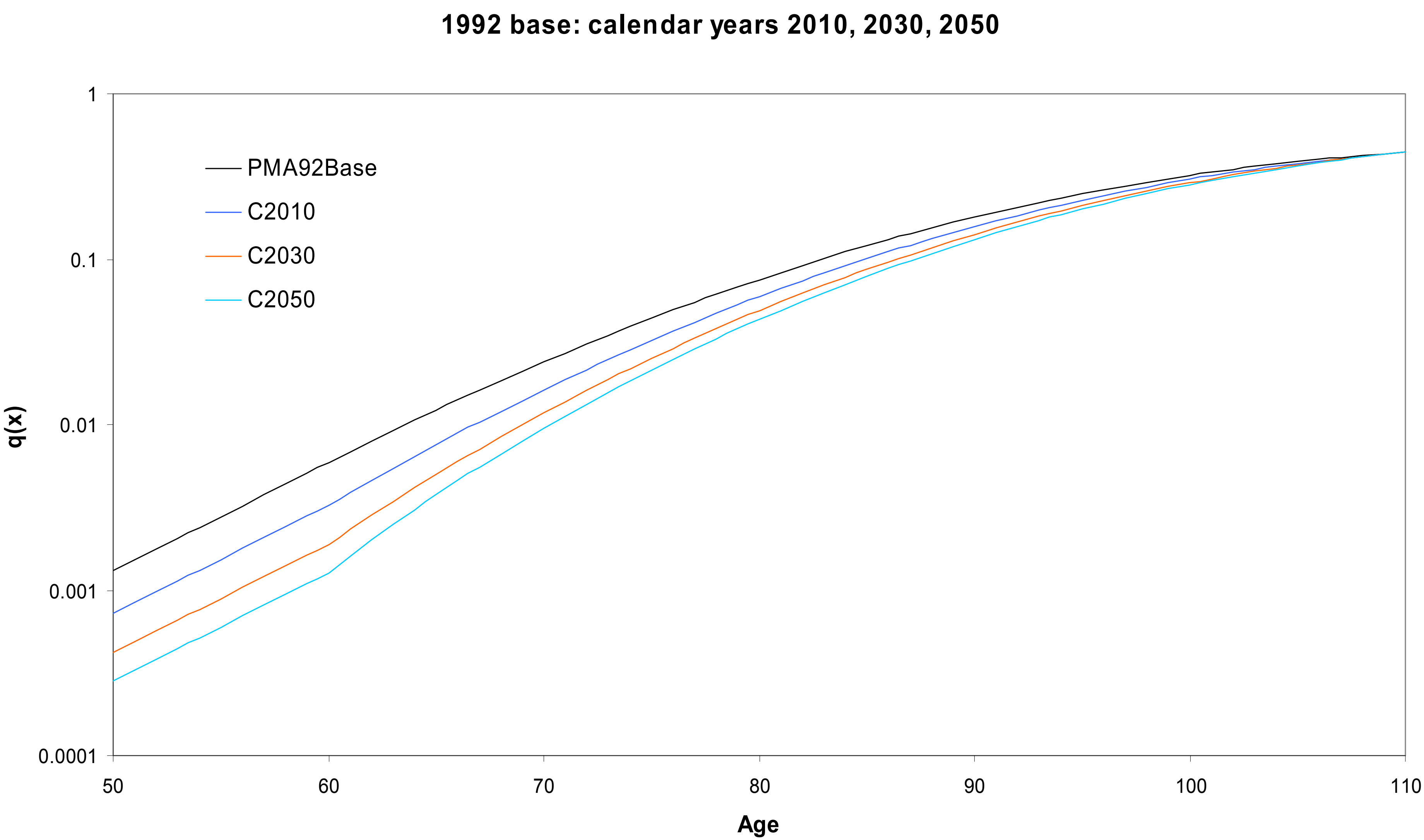
$$= k = 0.29 \qquad x > 110$$

$$
\alpha(x) = \begin{cases} c = 0.13 & x < 60 \\ 1 + (1 - c) \times (x - 110)/50 & 60 \le x \le 110 \\ 1 & x > 110 \end{cases}
$$

More complicated than 1980 method



1992 base: year of birth 1930, 1950, 1970
PMA92Base
B1930
B1950
B1970
q(x)
1
0.1
0.01
0.001
0.0001
50
60
70
80
90
100
110
Age

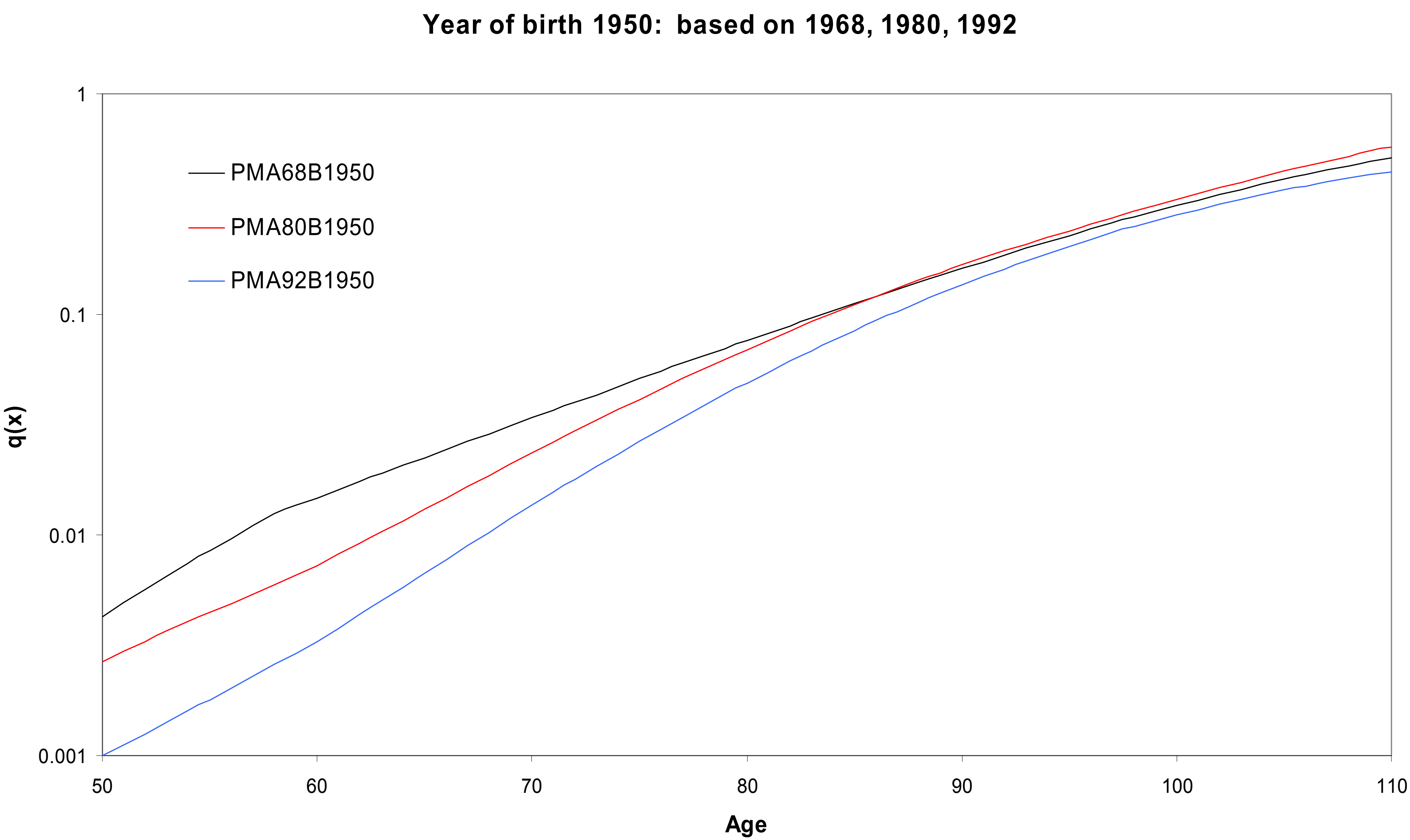

1992 base: calendar years 2010, 2030, 2050
PMA92Base
C2010
C2030
C2050
q(x)
1
0.1
0.01
0.001
0.0001
50
60
70
80
90
100
110
Age

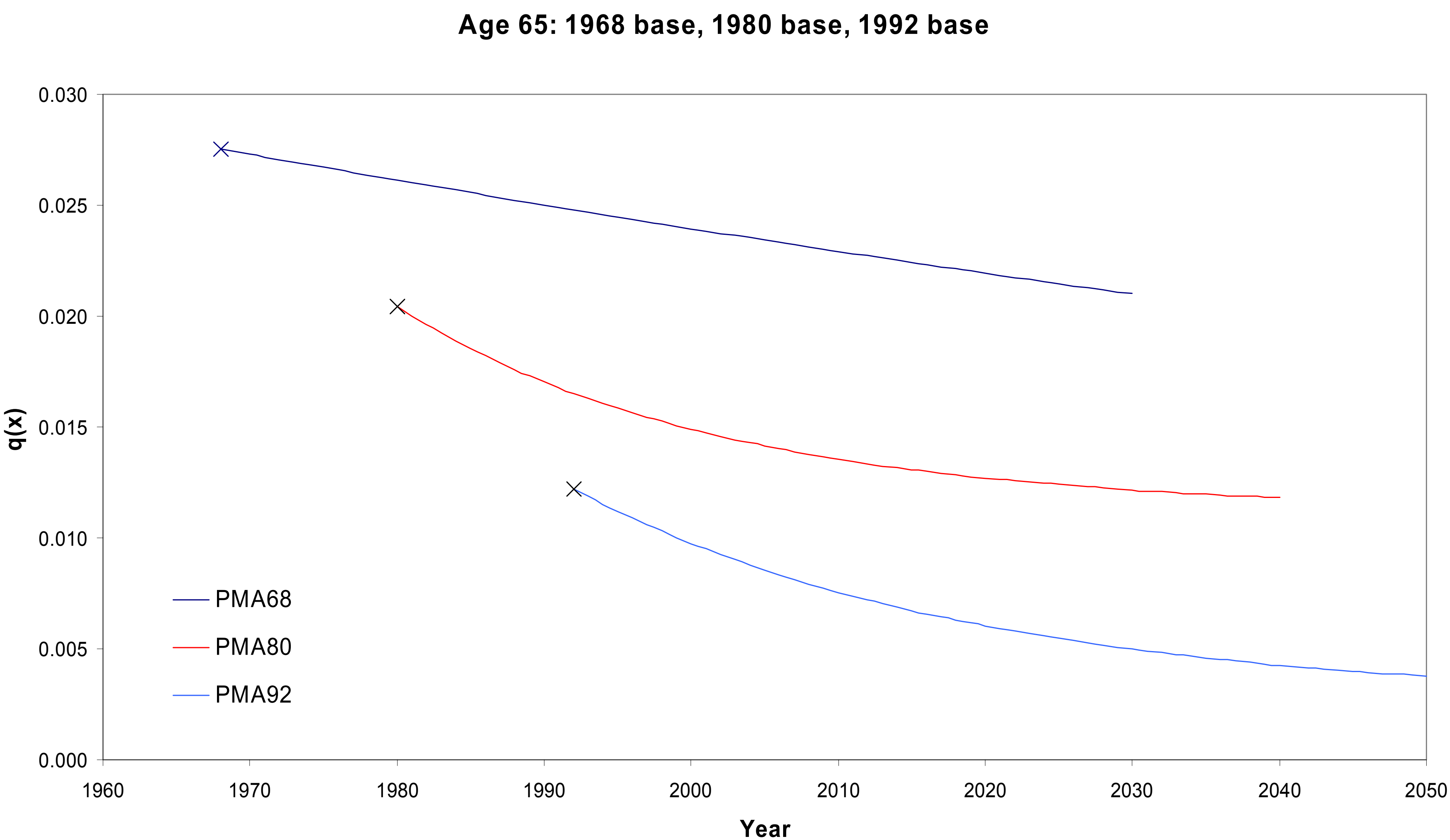
Similar reductions to 1980 projection

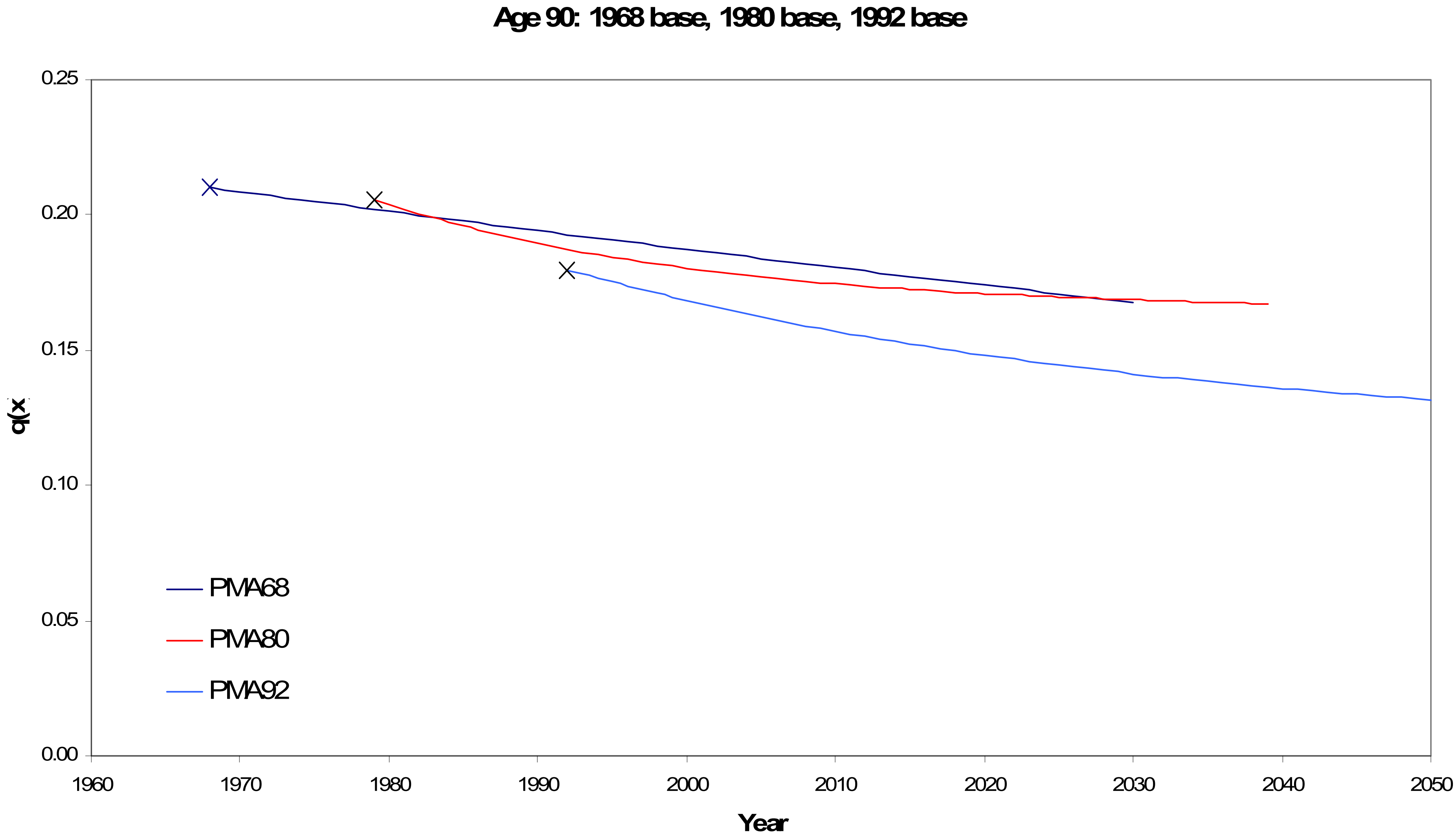
- **1 year in 6 at age 65**
- **1 year in 9 at age 90**

Now about

- **1 year in 5 at age 65**
- **1 year in 9 at age 90**


Year of birth 1950: based on 1968, 1980, 1992
PMA68B1950
PMA80B1950
PMA92B1950
q(x)
1
0.1
0.01
0.001
50
60
70
80
90
100
110
Age


Age 65: 1968 base, 1980 base, 1992 base
0.030
0.025
0.020
0.015
0.010
0.005
0.000
q(x)
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
Year
PMA68
PMA80
PMA92



Age 90: 1968 base, 1980 base, 1992 base
0.25
0.20
0.15
0.10
0.05
0.00
q(x
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
Year
PMA68
PMA80
PMA92

Annuity values a(x) at 4%

PMA68Base	9.51
PMA80Base	10.03
PMA92Base	11.22

Age 65 in base year:

PMA68B1903	**9.66**	**(cf 9.51)**
PMA80B1915	**10.47**	**(cf 10.03)**
PMA92B1927	**11.79**	**(cf 11.79)**

Born:	1930	1950	1970
PMA68	10.13	10.48	10.82
PMA80	11.02	11.39	11.54
PMA92	11.96	12.83	13.35

Projecting

e.g. using 1980Base and 1992Base

Two ideas:

improvements depend only on age

improvements depend on year of birth

1 **Take ratio $r(x) = q(x,92)/q(x,80)$**

Calculate using ratio for same age:

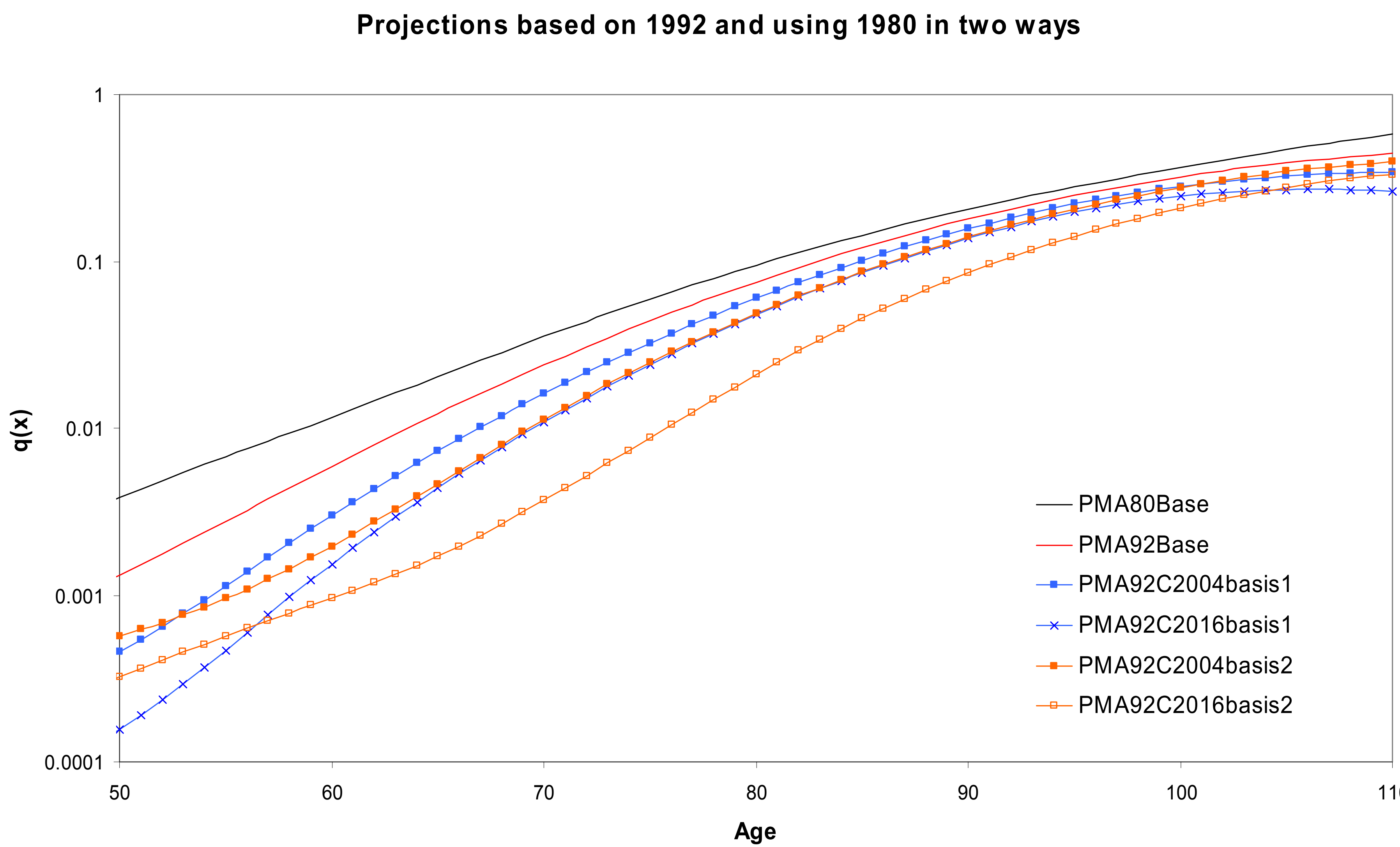
$$q(x,2004) = q(x,92)*r(x)$$

$$q(x,2016) = q(x,2004)*r(x)$$

2 **Calculate using ratio for year of birth:**

$$q(x,2004) = q(x,92) \times r(x - 12)$$

$$q(x,2016) = q(x,2004) \times r(x - 24)$$


Projections based on 1992 and using 1980 in two ways
q(x)
1
0.1
0.01
0.001
0.0001
50
60
70
80
90
100
110
Age
PMA80Base
PMA92Base
PMA92C2004basis1
PMA92C2016basis1
PMA92C2004basis2
PMA92C2016basis2

More modern methods under consideration

e.g. penalised splines (P-splines)

See http://www.actuaries.org.uk

Recent paper in

http://www.actuaries.org.uk/files/pdf/sessional/sm20051024.pdf

Also search under "CMI"

Now an emphasis on uncertainty of forecasts

Need “confidence intervals”

Many methods suggested

- **Lee-Carter**
- **Lee and Yang (different Lee)**
- **P-splines**
- **Andrew Smith**
- **Time series**

Example simple model, same for all ages:

$$q(x,t) = qF(x,t) \times \exp(X(t) - \mathrm{var}[X(t)]/2)$$

qF(x,t) is central (mean) projection

Var[X(t)]/2 needed to keep mean unchanged

X(t) defined in three ways

1/ X(t) is random walk:

$$X(t) = X(t-1) + Z(t)$$

$$Z(t) \sim N(0,\sigma^2)$$

2/ X(t) is sum of autoregressive changes:

$$X(t) = X(t-1) + Y(t)$$

$$Y(t) = a.Y(t-1) + Z(t)$$

$$Z(t) \sim N(0,\sigma^2)$$

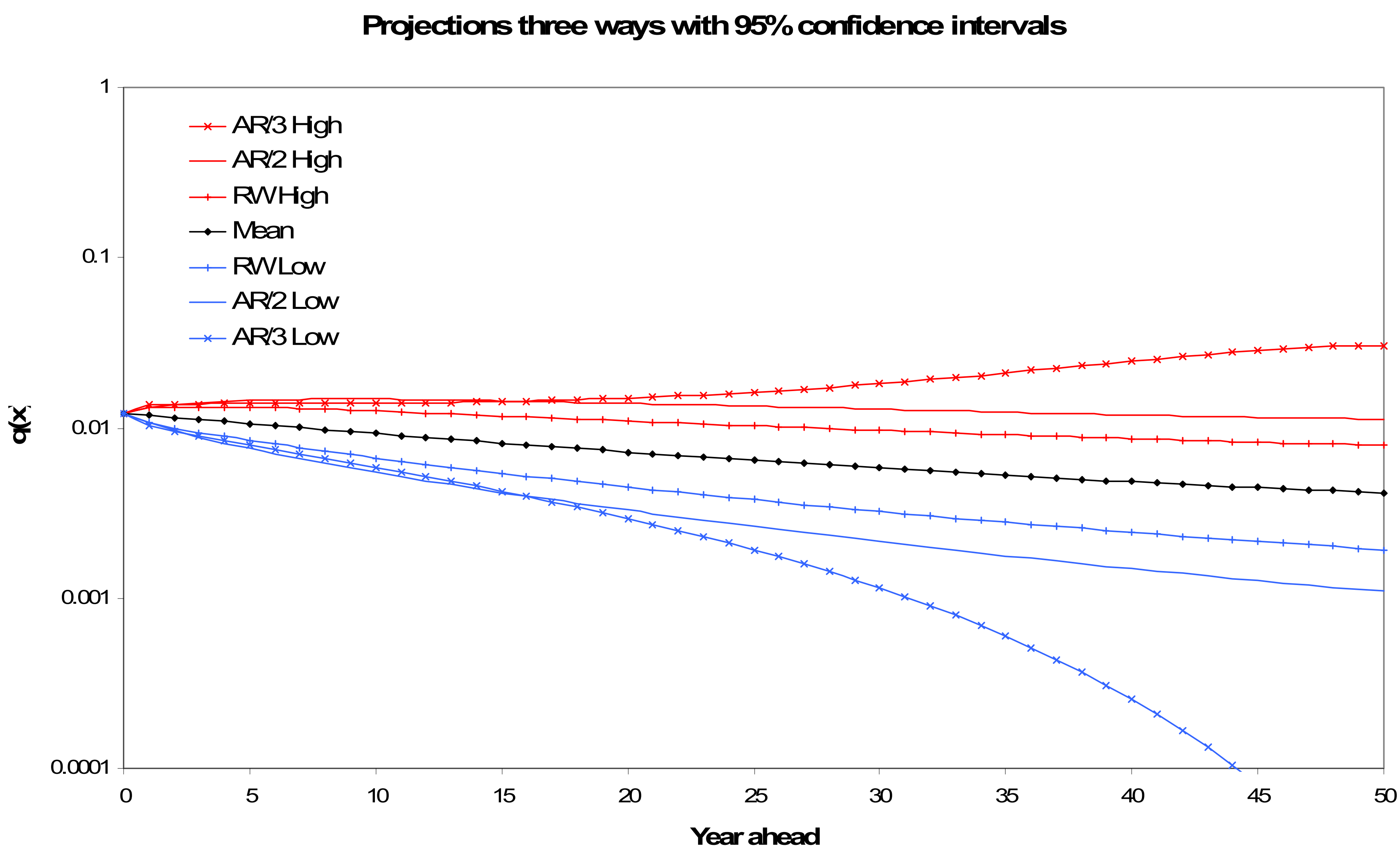
If $a = 0$ then $Y(t) = Z(t)$ and same as 1

3/ X(t) is autoregressive:

$$X(t) = a.X(t-1) + Z(t)$$

$$Z(t) \sim N(0,\sigma^2)$$

If a = 0 then again same as 1


Projections three ways with 95% confidence intervals
AR/3 High
AR/2 High
RW High
Mean
RW Low
AR/2 Low
AR/3 Low
1
0.1
0.01
0.001
0.0001
q(x)
0
5
10
15
20
25
30
35
40
45
50
Year ahead

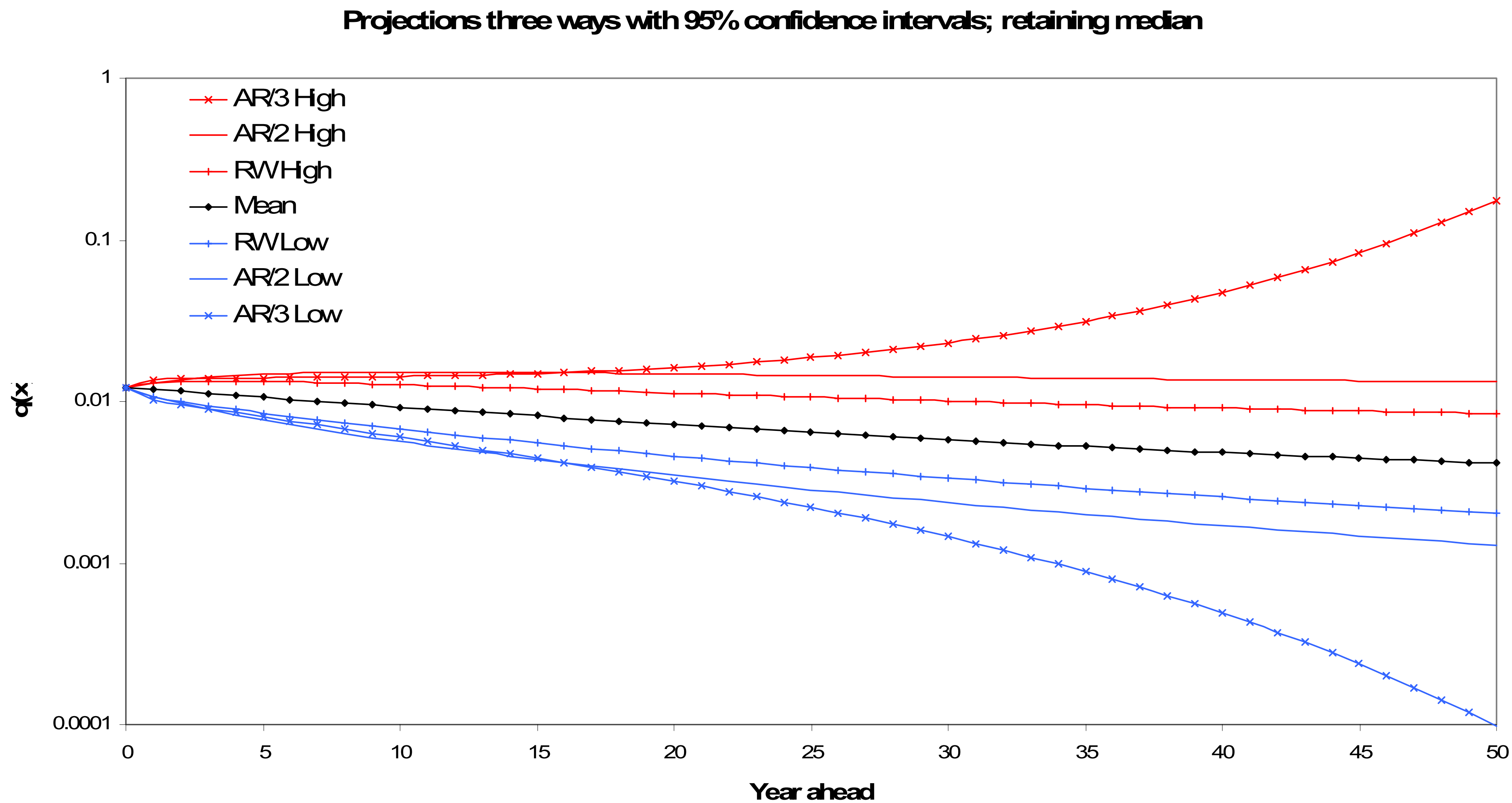
Much greater uncertainty with some models than with others

In fact much easier for mortality rates to go up than down:

wars, infections, natural disasters

So perhaps keeping the mean the same as the projection is not so good

Perhaps keep the median the same



Projections three ways with 95% confidence intervals; retaining median
AR/3 High
AR/2 High
RW High
Mean
RW Low
AR/2 Low
AR/3 Low
1
0.1
0.01
0.001
0.0001
q(x
0
5
10
15
20
25
30
35
40
45
50
Year ahead

Still much to be done about stochastic mortality

End